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Low-light Image Enhancement Based on Variational Retinex Model

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ABSTRACT

The low-light image enhancement plays a crucial role in computer vision and multimedia applications. However, it is still a challenging task, as the degraded images reduce the visual naturalness and visibility. To address this problem, we build a novel variational Retinex model to accurately estimate the illumination and reflectance components. The illumination and reflectance are jointly updated by alternating optimization algorithm. Experimental results on several public datasets demonstrate that the proposed method outperforms the state-of-the-art methods in Retinex decomposition and illumination adjustment.

Keywords: Low-light image enhancement, variational Retinex model, illumination adjustment

1. INTRODUCTION

Taken in low-light conditions such as darkness and nighttime, low-light images suffer from unpleasant visual aesthetics, enormous noise, low contrast and color distortions. These degraded images will affect the performance of subsequent computer vision algorithms. Besides, with the prevalence of portable imaging devices, the demand for high-quality images with clear details and satisfied brightness becomes extremely imperative. Therefore, the demand of effective yet robust model for low-light image enhancement under various realistic scenes is highly urgent. The methods of low-light enhancement can be generally divided into three categories, namely histogram equalization-based methods¹, Retinex decomposition-based methods², and deep learning-based methods³. The Histogram equalization (HE)-based methods enhance the visibility of low-light images by flattening the histogram via stretching the corresponding dynamic range of the intensity^{4, 5}. HE-based methods can be further classified into global HE-based methods and local HE-based methods. Although these methods are effective for dynamic range image enhancement, the enhanced image often exhibits unnatural details.

The Retinex decomposition-based methods enhance low-light images by image decomposition. These methods decompose the images into two components, namely reflectance and illumination. Then, these two components are further processed to obtain the enhanced results. The single-scaled Retinex (SSR)⁶ method and multi-scaled Retinex (MSR)⁷ method are the pioneering works in this field. Sequential methods consider both the illumination and reflectance layers to improve the performance⁸. However, it's inherently an ill-posed problem to estimate illumination and reflectance components from a single image. In order to make the problem trackable, some attempts transform the illumination or reflectance decomposition into a statistical reasoning problem and seek the optimal solutions by proposing different priors for illumination and reflectance and defining variational optimization⁹.

The learning-based methods model the feature maps from the high visual quality images to enhance the low-light images. Lore³ *et al.* first enhanced the low-light images by the stacking sparse auto-encoders. Subsequently, varied networks and diversified losses were proposed¹⁰. Besides, adversarial learning was introduced to obtain visual attributes beyond traditional metrics^{11, 12}. Jiang¹¹ *et al.* proposed an EnlightenGAN to get rid of the construction of pairwise datasets. Although the deep learning-based approaches have achieved remarkable achievements in the domain of low-light image enhancement, the enormous computational burden in practical application and the complex structure of the model limits their popularity on mobile devices. Moreover, the learning-based methods rely heavily on great deal of high-quality images.

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In this paper, we conduct an effective method to accomplish the structure and texture estimation during the Retinex decomposition. The proposed model is based on two highly correlated hypotheses, *i.e.*, the illumination component should be piece-wise smooth, while the reflectance component should contain as much detail as possible. Based on the structure and texture diagrams, we build a variational Retinex model to accurately estimate the illumination and reflectance components. Experiments on several challenging benchmarks verify the effectiveness of the proposed model in both subjective and objective evaluations.

2. METHODOLOGY

2.1 Retinex Model

Retinex theory¹³ postulates that the input low-light image $O \in \mathbb{R}^{n \times m}$ can be represented as the product of illumination $L \in \mathbb{R}^{n \times m}$ and reflectance $R \in \mathbb{R}^{n \times m}$ as $O = L \odot R$. The symbol \odot means the element-wise multiplication. The decomposed components can be converted back by estimating them alternatively by $L = O \oslash R$ and $R = O \oslash L$, where \oslash represents the element-wise division. Retinex theory introduces a valuable derivative property¹⁴, *i.e.*, the larger derivative value in the image is usually due to the variation of the reflectance component, while the smaller derivative value may be due to the smooth distribution of the illumination. According to the properties of the image in the gradient field, previous variational Retinex methods generally utilize a variational objective function to estimate the illumination and reflectance components¹⁵. The objective function is formulated as

$$\min_{L,R} \|O - L \odot R\|_F^2 + \mathcal{N}_1(L) + \mathcal{N}_2(R), \quad (1)$$

where \mathcal{N}_1 and \mathcal{N}_2 are regularization terms.

2.2 Structure and Texture Estimator

The regularization terms \mathcal{N}_1 and \mathcal{N}_2 in Eq. (1) can extract the structure and texture maps by distinguishing the difference in the distribution of gradients between the illumination and reflectance components. The smaller gradient is due to the smooth in illumination¹⁴, while the larger gradient is due to the variations in reflectance. These local derivatives can reflect the content structure or texture through exponential growth or decay. For this purpose, we adopt a novel relative total variation to construct the structure and the texture prior matrix. The structure prior is to enforce the spatial smooth on illumination layer while preserve the main structure in illumination layer.

The formulation of the structure prior is given as

$$S(O_p) = \sum_{q \in \mathcal{R}_p} \left(\frac{\mathcal{G}_\sigma * |\nabla_x O_q|}{|\mathcal{G}_\sigma * \nabla_x O_q| + \epsilon} + \frac{\mathcal{G}_\sigma * |\nabla_y O_q|}{|\mathcal{G}_\sigma * \nabla_y O_q| + \epsilon} \right)^{\gamma_s} \quad (2)$$

The proposed texture prior is to enforce the reflectance component to be piece-wise continuous. The texture prior is formulated as

$$T(O_p) = \sum_{q \in \mathcal{R}_p} \frac{1}{\left(\frac{\mathcal{G}_\sigma * |\nabla_x O_q|}{|\mathcal{G}_\sigma * \nabla_x O_q| + \epsilon} + \frac{\mathcal{G}_\sigma * |\nabla_y O_q|}{|\mathcal{G}_\sigma * \nabla_y O_q| + \epsilon} + \epsilon \right)^{\gamma_t}} \quad (3)$$

where $\epsilon = 0.001$ and $\epsilon = 0.005$. γ_s and γ_t are the structure and texture perception coefficients. Where $\nabla_{x/y}$ is partial derivative in horizontal or vertical directions. The symbol $*$ is a convolutional operator. \mathcal{G}_σ is the standard Gaussian kernel with window size $\sigma = 3$. \mathcal{R}_p is a rectangular region centered on the pixel p , and q is the pixel belongs to \mathcal{R}_p .

2.3 Proposed Model

The proposed model is formulated as

$$\operatorname{argmin}_{L,R} \|O - L \odot R\|_F^2 + \alpha \|S \odot \nabla L\|_F^2 + \beta \|T \odot \nabla R\|_F^2 + \lambda \|L - B\|_F^2, \quad (4)$$

where α , β and λ control the importance of different terms in the objective function. $\|O - L \odot R\|_F^2$ constrains the fidelity between the observed image O and the reconstructed image $L \odot R$. $\|\mathcal{S} \odot \nabla L\|_F^2$ and $\|\mathcal{T} \odot \nabla R\|_F^2$ correspond the structure map of illumination component and texture map of texture map. $\|L - B\|_F^2$ minimizes the distance between the estimated illumination L and the initial illumination B .

The formulations of the second and third terms in Eq. (4) are denoted as

$$\|\mathcal{S} \odot \nabla L\|_F^2 = s_x \|\nabla_x L\|_F^2 + s_y \|\nabla_y L\|_F^2, \quad (5)$$

$$\|\mathcal{T} \odot \nabla R\|_F^2 = t_x \|\nabla_x R\|_F^2 + t_y \|\nabla_y R\|_F^2, \quad (6)$$

where

$$s_{x/y} = \left(\frac{\mathcal{G}_\sigma * |\nabla_x L|}{|\mathcal{G}_\sigma * \nabla_x L| + \epsilon} \right)^{\gamma_s}, \quad (7)$$

$$t_{x/y} = \frac{1}{\left(\frac{\mathcal{G}_\sigma * |\nabla_x R|}{|\mathcal{G}_\sigma * \nabla_x R| + \epsilon} + \epsilon \right)^{\gamma_t}}. \quad (8)$$

2.4 Optimization Algorithm

Denote that L_k and R_k are the illumination and reflectance components at the k -th iteration ($k=0, 1, 2, \dots, K$), respectively. K is the maximum number of iterations. Two separated sub-problems are iteratively cycled through. We illuminate the solutions of the k -th iteration to the sub-problems as follows.

- 1) **L Sub-problem.** Neglecting the terms unrelated to L and initializing $L_0=O$, the optimization problem to L is formulated as

$$L_{k+1} = \operatorname{argmin}_L \|O - L \odot R_k\|_F^2 + \alpha(s_x \|\nabla_x L\|_F^2 + s_y \|\nabla_y L\|_F^2) + \lambda \|L - B\|_F^2. \quad (9)$$

To solve Eq. (9), the loss function to the matrix notation form is rewritten as

$$L_{k+1} = (L \odot R_k - O)^T (L \odot R_k - O) + \alpha(L^T D_x^T S_x D_x L + L^T D_y^T S_y D_y L) + \lambda(L - B)^T (L - B), \quad (10)$$

where D_x and D_y are the Toeplitz matrices in horizontal and vertical directions. $S_x = \operatorname{diag}(s_x)$ and $S_y = \operatorname{diag}(s_y)$. Then, the solution to Eq. (9) is

$$L_{k+1} = \frac{R_k^T O + \lambda B}{R_k^T R_k + \alpha(D_x^T S_x D_x + D_y^T S_y D_y) + \lambda I} \quad (11)$$

where I is an identity matrix.

- 2) **R Sub-problem.** We initialize the $R_0 = O \oslash L_1$ and update R while fixing L . The terms unrelated to R are neglected, and the following optimization problem is derived.

$$R_{k+1} = \operatorname{argmin}_R \|O - L_{k+1} \odot R\|_F^2 + \beta(t_x \|\nabla_x R\|_F^2 + t_y \|\nabla_y R\|_F^2). \quad (12)$$

The solution to Eq. (12) is similar to Eq. (11). Rewrite the loss function to the matrix notation norm and get the solution to Eq. (13):

$$R_{k+1} = \frac{L_{k+1}^T O}{L_{k+1}^T L_{k+1} + \beta(D_x^T T_x D_x + D_y^T T_y D_y)}. \quad (13)$$

The cycled optimization continues until the convergence conditions¹⁶ are satisfied, or the iterations reaches a pre-defined threshold. The summary of the optimization method for the proposed model is demonstrated in Figure. 1.

Algorithm 1: The optimization of the proposed model

Input: Observed image O , parameters $\gamma_s, \gamma_t, \alpha, \beta$ and λ , maximum iterations K and stopping parameters δ .

```
1 Initializing  $L_0$  and  $R_0$ , and setting the structure and texture weight matrices  $\mathcal{S}_0$  and  $\mathcal{T}_0$ .
2 for  $k = 1 : K$  do
3   1. Compute structure weight  $\mathcal{S}_{k+1}$  by Eq. (7)
4   2. Update  $L_{k+1}$  by Eq. (11)
5   3. Compute texture weight  $\mathcal{T}_{k+1}$  by Eq. (8)
6   4. Update  $R_{k+1}$  by Eq. (13)
7   if  $\|L_{k+1} - L_k\|_F / \|L_k\|_F \leq \delta$  or  $\|R_{k+1} - R_k\|_F / \|R_k\|_F \leq \delta$  then
8     Stop Updating
9     else
10      Continue
11    end
12  end
13 end
```

Output: Estimated illumination and reflectance components.

Figure 1. The summary of the optimization method for the proposed model

2.5 Illumination Adjustment

After obtaining the enhanced components of illumination L , we adopt Gamma correction^{17,18}, $\hat{L} = L^{\frac{1}{\gamma}}$, to adjust the illumination component. Then, the enhanced result \hat{O} is generated by $\hat{O} = R \odot \hat{L}^{\frac{1}{\gamma}}$, where γ is empirically set to 2.2^{19,20}. Finally, the enhanced image is reversed from HSV to RGB to get the final result.

3. EXPERIMENTAL RESULTS AND ANALYSIS

3.1 Experiment Settings and Implementation Details

The experiments are performed on a PC with an Intel i5-10400 CPU, 2.90GHz and 16GB memory. We set the parameters as $\gamma_s = 1.0$, $\gamma_t = 0.75$, $\alpha = 0.001$, $\beta = 0.0001$, $\delta = 0.005$, and $\lambda = 0.25$. For a fair comparison, the results of the compared methods come either from the original papers or reproduced by the official codes. The proposed is compared with 6 SOTA methods, including Dong²¹, CVC²², SSR⁶, NPE²³, MF²⁴, and LIME²⁵. The comparisons are conducted on 6 benchmarks, *i.e.*, LIME²⁵, DICM²⁶, MEF²⁷, NPE²³, LOL²⁸, LOE²⁹.

3.2 Retinex Decomposition

The performance of the proposed model on image decomposition is depicted in Figure. 2. The top row is the illumination component and the bottom row is the reflectance component. It shows that the illumination layer decomposed by SSR⁶ in Figure. 2 (a) is suffered from serious pixel dislocation, which will introduce the "ghost effect". LIME²⁵ in Figure. 2 (b) can obtain reasonable illumination map, but the reflectance fails to capture the details. Comparatively speaking, the proposed model can enhance the spatial piece-wise smoothness while preserving the structural information. Also, the image details can be faithfully preserved across the whole reflectance component.

3.3 Qualitative Evaluation

The qualitative results are depicted in Figure. 3. Some observations can be derived as follows. First, the HE-based method, *e.g.*, CVC²², tends to preserve the details in image but cannot improve the brightness effectively. For example, in Figure. 3 (c), the brightness is barely enhanced compared with the original image. Second, the result based on the SSR⁶ has serious artifacts, *e.g.*, unrealistic edges, strongly boosted noise, and color distortion, as shown in Figure. 3 (d). Third, the methods of LIME²⁵ and Dong are effective in improving the image brightness, but they are suffered from artifacts such as over-enhancement or noise amplification. For instance, in Figure. 3 (b) and Figure. 3 (e), the edges of plant leaves are slightly blurred. Comparatively speaking, the NPE²³ and our method achieve satisfying visual quality in all the tested images, but our results are relatively more natural.

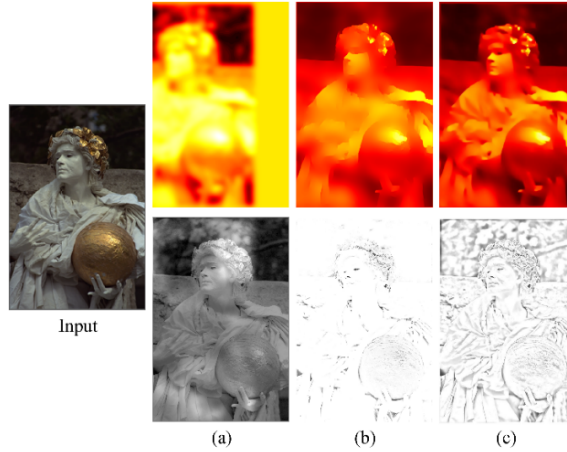


Figure 2. Decomposition results of the image "Statue". The first row are the estimated illumination maps and the second row are the estimated reflectance maps. (a) SSR⁶. (b) LIME²⁵ (c) Ours.

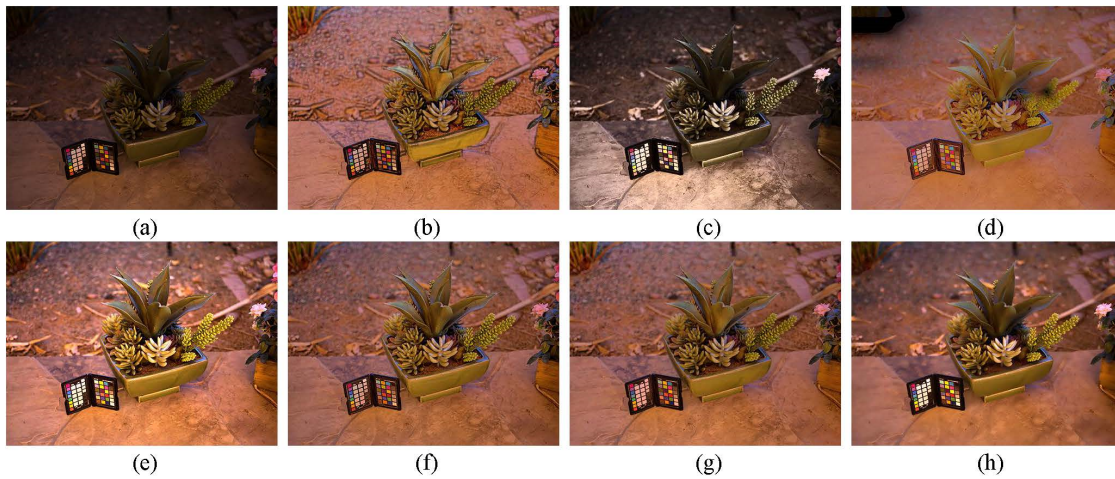


Figure 3. Qualitative evaluation with different methods. (a) Input (b) Dong²¹ (c) CVC²² (d) LDR³¹ (e) LIME²⁵ (f) MF²⁴ (g) NPE²³ (h) Ours.

Table 1. Quantitative comparisons in terms of NIQE. The best score is denoted in bold.

| Datasets Methods | LIME ²⁵ | DICM ²⁶ | NPE ²³ | LOE ²⁹ | LOL ²⁸ | MEF ²⁷ | Average |
|---------------------|--------------------|--------------------|-------------------|-------------------|-------------------|-------------------|---------------|
| Dong ²¹ | 4.3240 | 3.7747 | 4.2652 | 4.4903 | 3.8842 | 3.4191 | 4.0263 |
| CVC ²² | 3.7775 | 2.8245 | 3.1351 | 4.7309 | 5.1495 | 2.8315 | 3.7415 |
| SSR ⁶ | 4.2358 | 3.4375 | 3.1238 | 4.4214 | 4.1085 | 2.7745 | 3.6836 |
| NPE ²³ | 4.0751 | 3.2256 | 3.2540 | 4.3179 | 4.3507 | 2.7394 | 3.6605 |
| MF ²⁴ | 4.1301 | 3.1081 | 3.3805 | 4.8837 | 4.2423 | 2.7499 | 3.7491 |
| LIME ²⁵ | 4.5209 | 3.2488 | 3.4825 | 4.7246 | 4.1112 | 2.8096 | 3.8162 |
| Ours | 3.6212 | 3.0632 | 3.3322 | 4.6490 | 3.3730 | 3.6340 | 3.6121 |

3.4 Quantitative Evaluation

Considering that there is rare ground truth image in the dataset, we employ the NIQE indicator for quantitative evaluation. The NIQE gains the feature distribution via analyzing the dataset with high quality images. Then, it assesses the quality of an image by comparing the difference between the input image's feature distribution³² with the sample feature distribution. A lower NIQE value indicates higher quality of an image. Quantitative results in terms of NIQE are shown in Table 1. It shows that the proposed method performs best in LIME²⁵ and LOL²⁸ datasets. Meanwhile, it achieves an average score of 3.6121 which outperforms the SOTA methods.

4. CONCLUSION

In this paper, we propose a novel variational Retinex model for low-light image enhancement. We explore the priors of structure and texture on illumination and reflectance components. The key idea is to accurately estimate the structure and texture maps via analyzing the difference of gradient distribution in illumination and reflectance layers. The proposed model is solved by an alternative update algorithm. To demonstrate the effectiveness of the proposed model, experiments are carried on six public datasets. The subjective and objective evaluations demonstrate that the proposed method outperforms the state-of-the-art methods in both Retinex decomposition and illumination adjustment.

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